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ZNCC-based template matching using bounded partial correlation

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Abstract

This paper describes a class of algorithms enabling efficient and exhaustive matching of a template into an image based on the Zero mean Normalized Cross-Correlation function (ZNCC). The approach consists in checking at each image position two sufficient conditions obtained at a reduced computational cost. This allows to skip rapidly most of the expensive calculations required to evaluate the ZNCC at those image points that cannot improve the best correlation score *found so far*. The algorithms shown in this paper generalize and extend the concept of Bounded Partial Correlation (BPC), previously devised for a template matching process based on the Normalized Cross-Correlation function (NCC).

Keywords: Template matching; Bounded partial correlation; Normalized cross-correlation; NCC; ZNCC; BPC

1. Introduction

Template matching relies on calculating at each position of the image under examination a *correlation* or *distortion* function that measures the degree of similarity or dissimilarity to a template sub-image. Among the correlation/distortion functions proposed in literature, Normalized Cross-Correlation (NCC) and Zero mean Normalized Cross-Correlation (ZNCC) are widely used due to their

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robustness in template matching (Krattenthaler et al., 1994, Rosenfeld and Vanderburg, 1977a,b), motion analysis (Sun, 2002a) stereo vision (Sun, 2002b, Sun and Peleg, 2004, Faugeras et al., 1993) and industrial inspection (Tsai and Lin, 2003, Tsai et al., 2003). In fact, the normalization embodied into the NCC and ZNCC allows for tolerating linear brightness variations. Furthermore, thanks to the subtraction of the local mean, the ZNCC provides better robustness than the NCC (Faugeras et al., 1993, Tsai and Lin, 2003) since it tolerates uniform brightness variations as well. Since template matching based on the ZNCC or NCC can be very expensive, several *non-exhaustive* algorithms aimed at speeding-up the matching process have been developed (e.g. Krattenthaler et al., 1994; Rosenfeld and Vanderburg, 1977a,b; Barnea and Silverman, 1972). However, *non-exhaustive* algorithms do not explore the entire search space and therefore can be trapped into local maxima and yield a non-optimal solution. In this paper we propose an algorithm for ZNCC-based template matching that finds the same optimal solution as a full-search process and allows for significant computational savings. The algorithm is a generalization of the Bounded Partial Correlation (BPC) technique, previously devised only for NCC-based template matching (Di Stefano and Mattoccia, 2003a,b).

2. Generalization of the BPC technique to the ZNCC function

This section describes how to generalize the basic BPC technique based on the Cauchy-Schwarz inequality (Di Stefano and Mattoccia, 2003b) to a template matching process based on the ZNCC function. The novel technique will be referred to as Zero mean Bounded Partial Correlation (ZBPC).

Let I be the image under examination, of size $W \times H$ pixels, T the template sub-image, of size $M \times N$ pixels, and $I_c(x,y)$ the sub-image of size $M \times N$ located at pixel coordinates (x,y). Denoting as $\mu(T)$ and $\mu(I_c(x,y))$ the mean intensity value of T and $I_c(x,y)$, the Zero mean Normalized Cross-Correlation between T and I at pixel position (x,y) can be written as

$$\mathrm{ZNCC}(x,y) = \frac{\sum_{j=1}^{N}\sum_{i=1}^{M}[I(x+i,y+j)-\mu(I_c(x,y))]\cdot[T(i,j)-\mu(T)]}{\sqrt{\sum_{j=1}^{N}\sum_{i=1}^{M}[I(x+i,y+j)-\mu(I_c(x,y))]^2}\cdot\sqrt{\sum_{j=1}^{N}\sum_{i=1}^{M}[T(i,j)-\mu(T)]^2}}. \tag{1}$$

Denoting with $\psi(x,y)$ the *dot product* between $I_c(x,y)$ and T and with $\|\cdot\|$ the ℓ_2 norm, simple algebraical manipulations allows (1) to be written as

$$\mathrm{ZNCC}(x,y) = \frac{\psi(x,y) - M\cdot N\cdot\mu(I_c(x,y))\cdot\mu(T)}{\sqrt{||I_c(x,y)||^2 - M\cdot N\cdot\mu^2(I_c(x,y))}\cdot\sqrt{||T||^2 - M\cdot N\cdot\mu^2(T)}}. \tag{2}$$

Let us now call $\psi_Z(x,y)$ the numerator of (1) and split T and I_c into two portions (i.e. rows from $1,\ldots,n$ denoted with $|_1^n$ and rows $n+1,\ldots,N$ denoted with $|_{n+1}^N$) in order to express $\psi_Z(x,y)$ as a sum of two contributions:

$$\psi_Z(x,y) = \sum_{j=1}^{n}\sum_{i=1}^{M}[I(x+i,y+j)-\mu(I_c(x,y))] \cdot [T(i,j)-\mu(T)] + \sum_{j=n+1}^{N}\sum_{i=1}^{M}[I(x+i,y+j)-\mu(I_c(x,y))] \cdot [T(i,j)-\mu(T)] = \psi_Z(x,y)|_1^n + \psi_Z(x,y)|_{n+1}^N. \tag{3}$$

Starting from (3), two different bounding functions of the term $\psi_Z(x,y)$ can be devised. These yield two sufficient conditions to be used during the matching process in order to skip rapidly those pixel positions that cannot improve the current maximum ZNCC score.

2.1. The first sufficient condition

Denoting as ξ the sum of elements of a set and applying the Cauchy-Schwarz inequality to the latter term of (3), by means of simple manipulations we obtain an upper-bound of the term $\psi_Z(x,y)|_{n+1}^N$

$$\beta'_Z(x,y)|_{n+1}^N = \sqrt{(||T|||_{n+1}^N)^2 + (N-n)\cdot M\cdot\mu^2(T) - 2\cdot\mu(T)\cdot\xi(T)|_{n+1}^N} \Big/ \sqrt{(||I_c(x,y)|||_{n+1}^N)^2 + (N-n)\cdot M\cdot\mu^2(I_c(x,y)) - 2\mu(I_c(x,y))\cdot\xi(I_c(x,y))|_{n+1}^N}. \tag{4}$$

Calling $\eta_{z_{\max}}$ the maximum ZNCC score *found so far* and by replacing $\psi_Z(x,y)|_{n+1}^N$ with $\beta'_Z(x,y)|_{n+1}^N$ in (3) we obtain the following sufficient condition:

$$\frac{\psi_Z(x,y)|_1^n + \beta'_Z(x,y)|_{n+1}^N}{\sqrt{||I_c(x,y)||^2 - M\cdot N\cdot\mu^2(I_c(x,y))}\cdot\sqrt{||T||^2 - M\cdot N\cdot\mu^2(T)}} \leqslant \eta_{z_{\max}}. \tag{5}$$

2.2. The second sufficient condition

A second upper bounding function and associated sufficient condition can be obtained by algebraically manipulating the $\psi_Z(x,y)|_{n+1}^N$ term appearing in (3) before the application of the Cauchy-Schwarz inequality. Hence, we first rewrite $\psi_Z(x,y)|_{n+1}^N$ as

$$\psi_Z(x,y)|_{n+1}^N = \psi(x,y)|_{n+1}^N - \mu(T)\cdot\xi(I_c(x,y))|_{n+1}^N - \mu(I_c(x,y))\cdot\xi(T)|_{n+1}^N + (N-n)\cdot M\cdot\mu(T)\cdot\mu(I_c(x,y)). \tag{6}$$

Applying the Cauchy-Schwarz inequality to the dot product term $\psi(x,y)|_{n+1}^N$ in (6) we obtain the upper bound $\beta''_Z(x,y)|_{n+1}^N$

$$\beta''_Z(x,y)|_{n+1}^N = ||I_c(x,y)|||_{n+1}^N\cdot||T|||_{n+1}^N - \mu(T)\cdot\xi(I_c(x,y))|_{n+1}^N - \mu(I_c(x,y))\cdot\xi(T)|_{n+1}^N + (N-n)\cdot M\cdot\mu(T)\cdot\mu(I_c(x,y)) \tag{7}$$

that yields the sufficient condition

$$\frac{\psi_Z(x,y)|_1^n + \beta''_Z(x,y)|_{n+1}^N}{\sqrt{||I_c(x,y)||^2 - M\cdot N\cdot\mu^2(I_c(x,y))}\cdot\sqrt{||T||^2 - M\cdot N\cdot\mu^2(T)}} \leqslant \eta_{z_{\max}}. \tag{8}$$

2.3. The basic ZBPC algorithm

Starting from the sufficient conditions (5) and (8) we outline here the basic ZBPC algorithm. Let $n \in \{1, N-1\}$,

(1) Consider the next pixel position $(x,y) \in I$.
(2) Compute $\psi(x,y)|_1^n$, $\beta'_Z(x,y)|_{n+1}^N$ and $\beta''_Z(x,y)|_{n+1}^N$.
(3) If (5) or (8) holds go to step 1 else compute $\psi(x,y)|_{n+1}^N$.
(4) If $\mathrm{ZNCC}(x,y) > \eta_{z_{\max}}$ update $\eta_{z_{\max}}$ together with the current best matching position $(x_{\max}, y_{\max})$.
(5) Go to step 1.

2.4. Improving the basic ZBPC algorithm

Similar to BPC, ZBPC is a data dependent optimization technique that gets more effective as long as the degree of similarity between the template and the current best matching sub-image increases (i.e. $\eta_{z_{\max}}$ gets higher). This can be exploited to improve the basic algorithm according to one (or both) of the following two approaches.

- $\eta_{z_{\max}}$ can be initialized to the threshold value, η_{z_t}, used in most template matching processes in order to establish the minimum degree of similarity required to accept a match.
- Should an estimation of the best correlation score, $\hat{\eta}_{z_{\max}}$, be available (e.g. as a result of a non-exhaustive search), $\eta_{z_{\max}}$ can be initialized to $\hat{\eta}_{z_{\max}}$. To obtain $\hat{\eta}_{z_{\max}}$ we use a multiresolution approach, referred to as MZBPC, based on sub-sampling both I and T and then applying the basic ZBPC algorithm first at the lower resolution level and then, in the surroundings of the best matching position, at the higher resolution level. It is worth pointing out that though MZBPC incorporates an initial non-exhaustive step, the overall search process is exhaustive since the estimation of the best ZNCC score, $\hat{\eta}_{z_{\max}}$, is used only to initialize the current ZNCC maximum.

If both approaches are used then $\eta_{z_{\max}}$ is initialized to $\max\{\eta_{z_t}, \hat{\eta}_{z_{\max}}\}$.

With ZBPC and MZBPC, if at pixel position (x,y) one of the two sufficient conditions (5) and (8) holds, only a limited portion of the expensive $\psi_Z(x,y)|_1^n$ term needs to be calculated. Thus, to achieve computational savings it is mandatory that the two conditions can be calculated more rapidly than residual dot product term. First of all, terms $||T|||_{n+1}^N$, $\mu(T)$ and $\xi(T)|_{n+1}^N$ in (5) and (8) need to be computed only at start up. As for the terms that instead must be evaluated at each pixel position (x,y), we can observe that both conditions require the computation of $\psi_Z(x,y)|_1^n$ and the quantities $||I_c(x,y)|||_{n+1}^N$, $\mu(I_c(x,y))$ and $\xi(I_c(x,y))|_{n+1}^N$. The latter quantities can be computed very efficiently using incremental calculation techniques (Mc Donnell, 1981, Viola and Jones, 2001, Veksler, 2003) at a small and fixed cost that compares favorably to that required to attain the residual term $\psi_Z(x,y)|_{n+1}^N$. In fact, the calculation of $\psi_Z(x,y)|_{n+1}^N$ cannot be accelerated by incremental techniques and hence exhibits a computational cost growing with the template size.

3. Experimental results

Tables 1–3 provide the experimental results obtained with the three data sets shown respectively in Figs. 1–3.[1] The tables report, for the templates shown in the figures, the measured speed-ups and the percentage of image positions skipped by the ZBPC and MZBPC algorithms with respect to the standard ZNCC-based template matching algorithm. As for ZBPC, three different values of the sensitivity threshold have been used with each template (0%—no threshold—, 95% and 98% of the actual $\eta_{z_{\max}}$ score) in order to initialize $\eta_{z_{\max}}$. Conversely, in the MZBPC case $\eta_{z_{\max}}$ is initialized to the best score resulting from an initial coarse resolution search based on the sub-sampling factor S. All the considered algorithms were implemented in C and tested on a 3.06 GHz Pentium 4 processor running Linux.

The first column of Tables 1–3 shows that the basic ZBPC technique can speed-up the execution time of a ZNCC-based template matching process from 1.31 to 3.91. As shown in columns 2 and 3, further substantial computational savings can be achieved by deploying the sensitivity threshold, which is a quite standard parameter in a template matching process, to initialize the current ZNCC maximum. The tables point out also that in most cases MZBPC allows for obtaining a better speed-up than ZBPC, yielding factors ranging from 3.18 to 7.00. Although not discussed in this paper for the sake of brevity, we point out that with MZBPC the computational savings turn out to be practically independent from the position of the actual maximum ZNCC score within the image. The tables report also the percentages of

[1] Courtesy of the Vision and Autonomous Systems Center (VASC) at Carnegie Mellon University.

Table 1
Data set of Fig. 1: measured speed-ups and percentages of image positions skipped by ZBPC and MZBPC

I_P	ZBPC [0%]	ZBPC [95%]	ZBPC [98%]	MZBPC
T_{P1}	2.64 (76.1%)	3.20 (84.1%)	5.38 (99.4%)	5.20 (99.6%)
T_{P2}	3.91 (90.8%)	4.91 (97.1%)	5.01 (97.5%)	4.80 (97.6%)
T_{P3}	1.41 (36.5%)	3.44 (87.2%)	4.36 (94.5%)	4.50 (96.7%)

Parameters: $n/N = 0.18$ and for MZBPC $S = 4$.

Table 2
Data set of Fig. 2: measured speed-ups and percentages of image positions skipped by ZBPC and MZBPC

I_N	ZBPC [0%]	ZBPC [95%]	ZBPC [98%]	MZBPC
T_{N1}	2.29 (71.7%)	4.60 (99.3%)	4.67 (99.7%)	4.57 (99.7%)
T_{N2}	1.52 (43.6%)	2.34 (73.1%)	2.87 (83.4%)	3.18 (88.4%)
T_{N3}	1.31 (31.2%)	2.74 (81.5%)	3.52 (91.7%)	3.81 (95.4%)

Parameters: $n/N = 0.21$ and for MZBPC $S = 8$.

Table 3
Data set of Fig. 3: measured speed-ups and percentages of image positions skipped by ZBPC and MZBPC

I_C	ZBPC [0%]	ZBPC [95%]	ZBPC [98%]	MZBPC
T_{C1}	3.04 (76.4%)	4.47 (88.3%)	6.21 (95.3%)	6.48 (96.3%)
T_{C2}	2.79 (73.2%)	4.48 (88.4%)	5.89 (94.4%)	6.12 (95.3%)
T_{C3}	2.77 (72.5%)	4.86 (90.3%)	6.73 (96.7%)	7.00 (97.5%)

Parameters: $n/N = 0.12$ and for MZBPC $S = 6$.

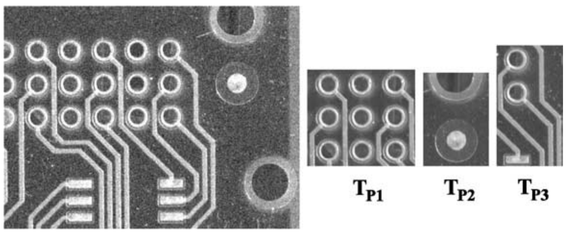

Fig. 1. Data set *Pcb*: image I_P and templates T_{P1}, T_{P2} and T_{P3}.

image positions skipped by ZBPC and MZBPC as a result of the anticipated detection of unsatisfactory matching candidates (i.e. the positions where one of the two sufficient conditions (5), (8) is satisfied). With the exception of template T_{P3} with no sensitivity threshold (see Table 1), the basic ZBPC approach allows for skipping always more than 72% of the image positions. As expected, the percentage of skipped positions gets notably better

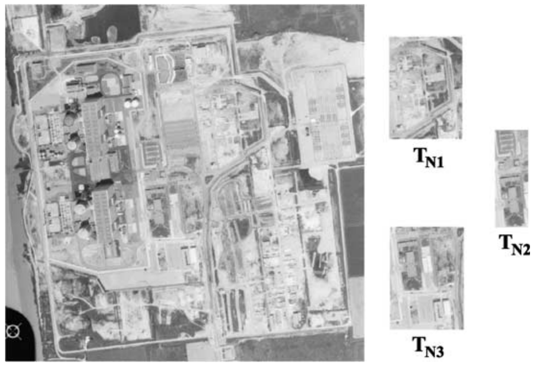

Fig. 2. Data set *Nuclear Plant*: image I_N and templates T_{N1}, T_{N2} and T_{N3}.

Fig. 3. Data set *Catalunya*: image I_C and templates T_{C1}, T_{C2} and T_{C3}.

as long as the sensitivity threshold gets higher and turns out to be the best with MZBPC. Excluding template T_{N2} (see Table 2), with MZBPC the sufficient condition (5) or (8) holds in more than 95% of image positions.

4. Conclusion and future work

We have described an algorithm for exhaustive template matching based on the direct computation of the ZNCC function. The algorithm generalizes the principle of the BPC technique, previously devised only for the NCC function, to the more robust and computationally expensive ZNCC function. The novel technique, referred to as ZBPC, is capable of rapidly rejecting mismatching positions thanks to two sufficient conditions based on the Cauchy-Schwarz inequality and calculated at a reduced computational cost. The computation of the whole algorithm can be efficiently performed thanks to computational schemes that require limited and fixed numbers of operations. Moreover, the basic ZBPC approach can be rendered significantly more effective by exploiting a correlation threshold and/or an initial estimation of the best ZNCC score. The experimental results indicate that the proposed algorithms can accelerate significantly an exhaustive ZNCC-based template matching process.

Preliminary experimental results show that function $\beta'_Z(x,y)$ tends to be more effective than $\beta''_Z(x,y)$ for small n values, while when n gets higher the effectiveness of the two bounding functions is reversed. Future work is aimed at embodying the proposed approach in a more sophisticated framework that shall allow for exploiting the predictable behavior of the bounding functions to deploy an optimal strategy for checking the two sufficient conditions.

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