An algorithm for efficient and exhaustive template matching

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Abstract. This paper proposes an algorithm for efficient and exhaustive template matching based on the Zero mean Normalized Cross Correlation (ZNCC) function. The algorithm consists in checking at each position a sufficient condition capable of rapidly skipping most of the expensive calculations involved in the evaluation of ZNCC scores at those points that cannot improve the best score *found so far*. The sufficient condition devised in this paper extends the concept of Bounded Partial Correlation (BPC) from Normalized Cross Correlation (NCC) to the more robust ZNCC function. Experimental results show that the proposed technique is effective in speeding up the standard procedure and that the behavior, in term of computational savings, follows that obtained by the BPC technique in the NCC case.

1 Introduction

Template matching consists in calculating at each position of the image under examination a function that measures the degree of similarity between a template and a portion of the image [1]. Normalized Cross-Correlation (NCC) and Zero mean Normalized Cross Correlation (ZNCC) are widely used similarity functions in template matching (e.g. [1–4]) as well as in motion analysis, stereo vision, industrial inspections and many other applications, since the normalization process embodied into the NCC and ZNCC allows for handling linear brightness variations. Furthermore, thanks to the subtraction of the mean intensity, the ZNCC function is even a more robust solution than the NCC since it can handle also uniform brightness variations. Since NCC and ZNCC are rather computationally expensive, several non exhaustive techniques aimed at reducing the computational cost have been proposed (e.g. [2–4]). Yet, *non-exhaustive* algorithms do not explore the entire search space and hence can be trapped into local maxima, thus yielding a non-optimal solution.

Conversely, in this paper we propose an algorithm that finds exactly the same optimal solution as a *brute force* ZNCC-based template matching process but at a significantly reduced computational cost. The proposed algorithm extends the concept of Bounded Partial Correlation (BPC), previously devised only for a template matching process based on the NCC [5, 6], to the ZNCC function.

2 A brief review of the BPC technique

Let I be the image under examination, of size $W \times H$ pixels, T the template, of size $M \times N$ pixels, and $I_c(x, y)$ the sub image of I at position (x, y) having the same size as the template (e.g. $I_c(x, y) = \{I(x+i, y+j) \mid i \in [1..M],\ j \in [1..N]\}$).

The Normalized Cross Correlation between the template T and the image I at position (x, y) is defined as:

$$NCC(x,y) = \frac{\sum_{j=1}^{N}\sum_{i=1}^{M} I(x+i, y+j) \cdot T(i,j)}{\sqrt{\sum_{j=1}^{N}\sum_{i=1}^{M} I^2(x+i, y+j)} \cdot \sqrt{\sum_{j=1}^{N}\sum_{i=1}^{M} T^2(i,j)}} \in [0,1] \tag{1}$$

The numerator of (1) represents the *dot product* between $I_c(x, y)$ and T, while in the remainder the ℓ_2 norms of $I_c(x, y)$ and T at the denominator will be denoted as $||I_c(x, y)||$ and $||T||$.

The BPC technique [5, 6] allows for speeding up an exhaustive template matching process based on the NCC by rapidly detecting unsatisfactory matching candidates. Such detection is achieved by evaluating a sufficient condition, obtained at a reduced computational cost, that relies on an upper bound, $\gamma(x, y, n)$, of the dot product term in (1):

$$\gamma(x,y,n) \geq \sum_{j=1}^{N}\sum_{i=1}^{M} I(x+i, y+j) \cdot T(i,j) \tag{2}$$

Let η_{max} represent the correlation maximum *'found so far'* during the matching process and that the point at coordinates (x, y) is currently under examination. The following inequality,

$$\frac{\gamma(x,y,n)}{||I_c(x,y)|| \cdot ||T||} < \eta_{max} \tag{3}$$

when holds, provides a sufficient condition for skipping the current position without carrying out the entire calculation of the computationally expensive dot product term since (3) guarantees that the current position cannot improve the η_{max} score.

An effective sufficient condition is obtained splitting both T and I_c into two parts, respectively denoted by rows [0..n] and [n+1..N] as showed in Figure 1, and, correspondingly, the dot product term into two partial terms:

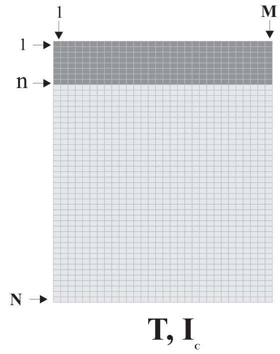

Fig. 1. *Splitting of the template T and sub-image I_c.*

$$\sum_{j=1}^{N}\sum_{i=1}^{M} I\left(x+i,y+j\right)\cdot T\left(i,j\right) = \\ \sum_{j=1}^{n}\sum_{i=1}^{M} I\left(x+i,y+j\right)\cdot T\left(i,j\right) + \sum_{j=n+1}^{N}\sum_{i=1}^{M} I\left(x+i,y+j\right)\cdot T\left(i,j\right) \tag{4}$$

Then, the upper bound $\gamma(x, y, n)$ is obtained by adding the first partial dot product term and an upper bound, $\beta(x, y, n)$, of second partial dot product term:

$$\gamma\left(x,y,n\right) = \sum_{j=1}^{n}\sum_{i=1}^{M} I\left(x+i,y+j\right)\cdot T\left(i,j\right) + \beta\left(x,y,n\right) \tag{5}$$

As shown in Figure 1, the index n in (5) determines the splitting of the dot product term into two partial terms. Applying the Cauchy-Schwarz inequality to the rightmost term of (4) yields to the bounding function $\beta(x, y, n)$:

$$\beta\left(x,y,n\right) = ||I_c\left(x,y\right)|||_{n+1}^{N} \cdot ||T|||_{n+1}^{N} \tag{6}$$

where $||I_c\left(x,y\right)|||_{n+1}^{N}$ and $||T|||_{n+1}^{N}$ represent the *partial ℓ_2 norms* of terms I_c and T within rows $(n + 1)$ and N.

By plugging (6) into (5) we obtain a sufficient condition (e.g. (3)) that allows for skipping unsatisfactory matching candidates. It relies on a portion of the dot product term and a bounding function $\beta(x, y, n)$ that can be calculated very efficiently using incremental computation schemes (i.e. *box-filtering* [7]), at the cost of a limited and fixed number of operations.

It is worth observing that, in the examined case of a single partition of T and I_c, the splitting procedure of the dot product term is mandatory since if the bounding function is defined over the whole area (e.g. case of n set to zero) then inequality (3) never holds.

3 Extension of the BPC technique to the ZNCC

This section describes how to extend the BPC technique based on the Cauchy-Schwarz inequality to the more robust and computationally expensive ZNCC function. The novel technique will be referred to as Extended Bounded Partial Correlation (EBPC).

Denoting with $\mu(T)$ and $\mu(I_c(x,y))$ the mean intensity values computed, respectively, on T and on $I_c(x,y)$, the Zero mean Normalized Cross Correlation between T and I at position (x,y) is defined as:

$$ZNCC(x,y) = \frac{\sum_{j=1}^{N}\sum_{i=1}^{M}\left[I(x+i,y+j) - \mu(I_c(x,y))\right]\cdot\left[T(i,j)-\mu(T)\right]}{\sqrt{\sum_{j=1}^{N}\sum_{i=1}^{M}\left[I(x+i,y+j) - \mu(I_c(x,y))\right]^2}\cdot\sqrt{\sum_{j=1}^{N}\sum_{i=1}^{M}\left[T(i,j)-\mu(T)\right]^2}}$$
$$= \frac{\sum_{j=1}^{N}\sum_{i=1}^{M} I(x+i,y+j)\cdot T(i,j) - M\cdot N\cdot\mu(I_c(x,y))\cdot\mu(T)}{\sqrt{||I_c(x,y)||^2 - M\cdot N\cdot\mu^2(I_c(x,y))}\cdot\sqrt{||T||^2 - M\cdot N\cdot\mu^2(T)}} \in [-1,1] \tag{7}$$

Similarly to the NCC case, let's split the template T and the sub-image I_c into two portions, as shown in Figure 1, and correspondingly the numerator of (7) into two terms:

$$\begin{aligned}&\sum_{j=1}^{N}\sum_{i=1}^{M}\left[I(x+i,y+j)-\mu(I_c(x,y))\right]\cdot\left[T(i,j)-\mu(T)\right] = \\ &\sum_{j=1}^{n}\sum_{i=1}^{M}\left[I(x+i,y+j)-\mu(I_c(x,y))\right]\cdot\left[T(i,j)-\mu(T)\right] + \\ &\sum_{j=n+1}^{N}\sum_{i=1}^{M}\left[I(x+i,y+j)-\mu(I_c(x,y))\right]\cdot\left[T(i,j)-\mu(T)\right]\end{aligned} \tag{8}$$

where, as usual, n represents the number of rows determining the two portions of template T and sub-image I_c.

The first term at the second member of (8), referred to as *partial correlation* (e.g. $P_c(x,y,n)$), may be written in a more convenient form as follows:

$$P_c(x,y,n) = \sum_{j=1}^{n}\sum_{i=1}^{M}\left[I\left(x+i,y+j\right)-\mu\left(I_c\left(x,y\right)\right)\right]\cdot\left[T\left(i,j\right)-\mu\left(T\right)\right] =$$
$$\sum_{j=1}^{N}\sum_{i=1}^{M} I\left(x+i,y+j\right)\cdot T\left(i,j\right)+$$
$$- M\cdot n\cdot\left[\mu(T)\cdot\mu(I_c(x,y))\Big|_1^n + \mu(I_c(x,y))\cdot\mu(T)\Big|_1^n + \mu(I_c(x,y))\cdot\mu(T)\right] \tag{9}$$

where $\mu(I_c(x,y))|_1^n$ and $\mu(T)|_1^n$ represent the *partial mean intensity values* between rows 1 and n referred, respectively, to the I_c and T term.

A bounding function of the numerator of the ZNCC function can be devised by applying the Cauchy-Schwarz inequality to the rightmost term in (8):

$$\beta_Z\left(x,y,n\right) = \sqrt{\sum_{j=n+1}^{N}\sum_{i=1}^{M}\left[I\left(x+i,y+j\right)-\mu(I_c(x,y))\right]^2}\cdot\sqrt{\sum_{j=n+1}^{N}\sum_{i=1}^{M}\left[T\left(i,j\right)-\mu(T)\right]^2} \tag{10}$$

Then, by simple algebraical manipulations:

$$\beta_Z\left(x,y,n\right) = \sqrt{(||T|||_{n+1}^N)^2 + (N-n)\cdot M\cdot\mu(T)\cdot[\mu(T) - 2\cdot\mu(T)\big|_{n+1}^N]}\cdot$$
$$\cdot\sqrt{(||I_c\left(x,y\right)|||_{n+1}^N)^2 + (N-n)\cdot M\cdot\mu(I_c(x,y))\cdot[\mu(I_c(x,y)) - 2\cdot\mu(I_c(x,y))\big|_{n+1}^N]} \tag{11}$$

Since function $\beta_Z(x,y,n)$ turns out to be an upper bound of a portion of the dot product term:

$$\beta_Z\left(x,y,n\right) \geq \sum_{j=n+1}^{N}\sum_{i=1}^{M}\left[I\left(x+i,y+j\right)-\mu\left(I_c\left(x,y\right)\right)\right]\cdot\left[T\left(i,j\right)-\mu\left(T\right)\right] \tag{12}$$

replacing the latter term of (8) with $\beta_Z\left(x,y,n\right)$ leads to the following upper bound of the numerator of the ZNCC function:

$$\gamma_Z\left(x,y,n\right) = P_c(x,y,n) + \beta_Z\left(x,y,n\right) \tag{13}$$

Finally, denoting as η_{zmax} the maximum ZNCC score *found so far*, (13) allows to obtain the following sufficient condition for safely rejecting unsatisfactory matching candidates:

$$\frac{\gamma_Z\left(x,y,n\right)}{\sqrt{||I_c\left(x,y\right)||^2 - M\cdot N\cdot\mu^2(I_c\left(x,y\right))}\cdot\sqrt{||T||^2 - M\cdot N\cdot\mu^2(T)}} \leq \eta_{z_{max}} \tag{14}$$

It is worth pointing out that with EBPC only a limited portion of the expensive dot product term needs to be calculated when the sufficient condition (14) holds. Viceversa, if the sufficient condition does not hold the dot product term has to be entirely computed.

Since the strength of the technique relies in avoiding this whole computation, in order to achieve effective performance improvements it is mandatory that the sufficient condition (14) could be calculated very efficiently and its outcome could hold as much as possible.

For this reason it is worth pointing out that the sufficient condition is made of terms (e.g. $||I_c(x, y)|||_{n+1}^{N}$, $\mu(I_c(x, y))$ and $\mu(I_c(x, y))|_{n+1}^{N}$) that can be efficiently computed using well-known incremental calculation techniques (e.g. [7]) requiring reduced and fixed overhead (i.e. 4 elementary operations for each term). This compares favorably with the dot product term since, conversely, this cannot be computed with incremental calculation techniques and hence its complexity grows with the template size resulting in the true bottleneck of the standard ZNCC-based algorithm

Finally, the remaining terms (e.g. $||T|||_{n+1}^{N}$, $\mu(T)$ and $\mu(T)|_{n+1}^{N}$) involved in the evaluation of (14) need to be computed and stored only once, at initialization.

4 Experimental results

This section provides experimental results concerned with the data set *Albert*, *Pcb3* and *Plants* shown in Figure 2. For each image, Table 1 shows the speed-up of the EBPC algorithm compared to the standard ZNCC-based template matching algorithm with four different initial values of $\eta_{z_{max}}$ (respectively 0%, 90%, 95% and 98% of the actual $\eta_{Z_{max}}$ score). For each test $\frac{n}{N}$ was set to 0.18. All the algorithms were implemented in C and the system used for the experimental results was a Linux PC with an AMD Thunderbird 900 MHz processor.

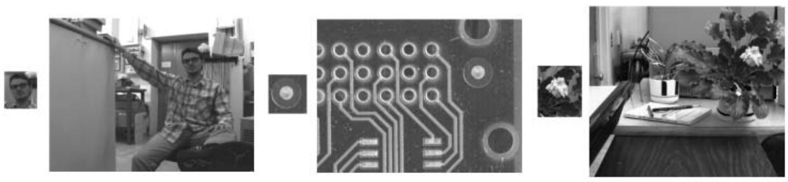

Fig. 2. *Data set: (Left) Albert (Center) Pcb3 (Right) Plants*

First column of Table 1 shows that the proposed EBPC technique is effective in increasing the computational efficiency of a ZNCC based template matching process by at least a factor of 1.9. Moreover, better results have been obtained by using a higher initial value of $\eta_{z_{max}}$. In fact, this allows to use a very effective sufficient condition starting from the initial image points examined during the search process.

Image	**EBPC** [0%]	**EBPC** [90%]	**EBPC** [95%]	**EBPC** [98%]
albert	1.90	2.69	2.74	2.78
pcb3	1.90	2.16	2.47	2.57
plants	2.40	2.56	3.00	3.32

Table 1. For the three images of Figure 2: measured speed-ups for the EBPC algorithm with four different initial values of $\eta_{z_{max}}$.

Table 2 shows the percentage of skipped points relatively to each algorithm and image presented in Table 1. The table shows that the basic EBPC technique allows for skipping more than 62% of the examined points. Moreover, as expected, when $\eta_{z_{max}}$ gets higher the number of skipped points increases significantly.

Image	**Points**	**EBPC** [0%]	**EBPC** [90%]	**EBPC** [95%]	**EBPC** [98%]
albert	48958	62.15 %	81.86 %	82.58 %	82.79 %
pcb3	97080	62.81 %	70.16 %	77.58 %	79.44 %
plants	113832	73.38 %	76.56 %	83.61 %	87.35 %

Table 2. For the three images of Figure 2: percentages of points skipped by the EBPC algorithm with four different initial values of $\eta_{z_{max}}$.

Finally, Table 3 and Table 4, show respectively the measured speed-up and the number of skipped points obtained in the case of the standard BPC algorithm compared to the the *brute force* NCC algorithm. It is worth observing that these results are similar to those obtained comparing the EBPC algorithm to the *brute force* ZNCC algorithm (Tables 1 and 2).

	BPC [0%]	**BPC** [90%]	**BPC** [95%]	**BPC** [98%]
albert	2.67	2.67	2.73	4.32
pcb3	2.09	2.09	2.09	2.10
plants	2.62	2.62	2.62	3.25

Table 3. For the three images of Figure 2: measured speed-ups for the original BPC algorithm with four different initial values of $\eta_{z_{max}}$.

5 Conclusions

We have described an efficient and exhaustive template matching algorithm based on direct computation of the ZNCC function. The algorithm extends the principles of the BPC technique, previously devised for the NCC, to the more robust ZNCC function. The proposed algorithm, referred to as EBPC, is capable

Image	**Points**	**BPC** [0%]	**BPC** [90%]	**BPC** [95%]	**BPC** [98%]
albert	48958	79.64 %	79.64 %	80.48 %	96.71 %
pcb3	67080	66.19 %	66.19 %	66.19 %	66.62 %
plants	113832	76.26 %	76.26 %	76.32 %	85.12 %

Table 4. For the three images of Figure 2: percentages of points skipped by the BPC algorithm with four different initial values of $\eta_{z_{max}}$.

of rapidly rejecting mismatching positions thanks to a sufficient condition based on the Cauchy-Schwarz inequality. The EBPC algorithm can be implemented very efficiently thanks the use of computational schemes that require limited and fixed numbers of operations.

Experimental results show that the EBPC algorithm compares favorably to the *brute force* ZNCC algorithm and that the behavior, in terms of measured speed-up, is similar to those obtained with the BPC technique in the NCC case.

A further improvement could be achieved using several elimination conditions based on increasing values of the parameter n. Besides, the implementation, currently under development, of the proposed algorithm with the parallel, SIMD-style, multimedia instructions available nowadays in most state-of-the-art microprocessors shall allow for further performance improvements.

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